The non-local action for the induced 2d supergravity. [1]

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Abstract

The two-dimensional simple supergravity is reexamined from the point of view of super-Weyl group cohomologies. The non-local form of the effective action of 2d supergravity which generalise the famous $R\frac{1}{\Box}R$ is obtained.

1 Introduction

The two-dimensional field-theoretical models are widely investigated at last two decades. The $2d$-models provides us an excellent laboratory for studying realistic, but much more complicated four-dimensional models. Indeed such phenomena as renormalization, asymptotic freedom, dimensional transmutation and others hold in integrable $2d$ models and can be described exactly. Finally from $2d$ models we can guess some information about structure of corresponding $4d$-models.

The $2d$-gravity takes an apparent place among $2d$-models. It happens not only by the fact that gravity is most mysterious part of quantum field theory, but also by its close relation to the string theory. After the famous work of A.M.Polyakov [1], which reduces non-critical strings to induced gravity, that theory became intensively studied, A.M.Polyakov [2] was compute most important indices of the theory in light-cone formulation, F.Distler, H.Kawai and F.David [3] was rederived that result in conformal gauge, then A.M.Polyakov, V.G.Kniznik and A.B.Zamolodchikov [4] discover that anomaly cancellation implies that target space dimension has to be fractal. Later it was shown that different regularization schemes are equivalent [10], it was found formulation of $2d$ induced supergravity, computed in Weyl-invariant regularization scheme too [5]. The holomorphic properties of the effective action has been studied in [6] and in [7]. In [8] made attempt to understand extrinsic geometry relation to the induced gravity. Authors of work [9] study relations between super-Virasoro and super-Weyl anomaly and construct super-Weyl invariant functional by adding non-local functional in order to cancell super-Virasoro anomaly. But till now the non-local expression for $2d$ induced supergravity analogous to famous $R\frac{1}{\Box}R$ remains unknown. The present work is called to cover that missing.

2 Computation

The graviton modes in gravity are described by traceless excitations of metric. The invariance of an action under general coordinate transformations implies the independence of the action from such a variations of the metric field. As well known in $2d$ general covariance is accompanied by local Weyl-symmetry, which implies the independence of action from the remaining variations of the metric: $g^{\alpha\beta}\delta g_{\alpha\beta}$ too. In other words, in $2d$ general covariant action actually is independent from the metric and $2d$ space-time is locally flat. However the quantum fluctuations spoil that

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symmetry and give rise the conformal anomaly: the variation of the effective action under such variations of metric is no longer equal to zero

$$g^{\alpha\beta}\frac{\delta W}{\delta g_{\alpha\beta}} = R \tag{1}$$

where R is the curvature of $2d$ space-time. It is easily to see that R defined as variational derivative of W satisfies to consistency condition: under Weyl rescalings

$$\delta g_{\alpha\beta} = g_{\alpha\beta}\delta\sigma \tag{2}$$

curvature transforms accordingly rule

$$\delta[\sqrt{g}R] = \sqrt{g}\Box\delta\sigma \tag{3}$$

So the second derivative of W by σ is symmetric:

$$\frac{\delta R(x)}{\delta\sigma(y)} = \frac{\delta R(y)}{\delta\sigma(x)}. \tag{4}$$

In $2d$ supergravity graviton's partner - gravitino describes by *spin* $3/2$ components of Rarita-Schwinger field. So, local supersymmetry implies independents of the action under *spin* $3/2$ variations of spin-vector field. Classically $2d$ supergravity also is locally flat: local supersymmetry in $2d$ is accompanied by invariance under variations of *spin* $1/2$ part of gravitino too

$$\delta\chi_\alpha = \gamma_\alpha\delta\lambda \tag{5}$$

That symmetry also is spoiled by quantum corrections

$$\frac{\delta W}{\delta\lambda} = \gamma_\alpha J^\alpha \tag{6}$$

where J^α is supercurrent of theory (superpartner of stress-tensor). Now its contraction with γ matrix is no longer equal to zero. In order to find super-Weyl anomaly, one can use integrability condition of the action: variational derivative of the curvature (Weyl-anomaly) by λ should be equal to the super-Weyl anomaly's derivative by σ. The λ-dependence of curvature comes from non-minimal term of spin-connection:

$$\omega_\alpha = -e^a_\alpha\frac{\epsilon^{\mu\nu}}{e}\partial_\mu e^a_\nu - 2i\bar{\chi}_\alpha\gamma_5\gamma^\mu\chi_\mu \tag{7}$$

Solving that equation, we can write down following set of anomaly equations for $2d$ supergravity computing in super-coordinate invariant regularization:

$$g^{\alpha\beta}T_{\alpha\beta} = R \tag{8}$$

$$\gamma^\alpha J_\alpha = -4i\frac{\epsilon^{\alpha\beta}}{e}\gamma_5 D_\alpha\chi_\beta \tag{9}$$

where $D_\alpha\lambda = \partial_\alpha\lambda + \frac{1}{2}\gamma_5\omega_\alpha\lambda$. It is easely to check that last Wess-Zumino condition is satisfied too, λ-derivative of super-Weyl anomaly (9) is symmetric in sense of eq.(4). Hence eqs.(8) and (9) define the consistent multiplet of supertrace anomaly for 2d simple supergravity. This expression for anomaly coincide with result of authors [11]. Transforming r.h.s. of this equations by finite Weyl and super-Weyl and multiplying them by $\delta\sigma$ and $\delta\lambda$ correspondingly and adding together we recognize the total variation of the effective action, which is Neveu-Schwarz action plus $R\sigma$ and $-4i\frac{\epsilon^{\alpha\beta}}{e}\bar{\lambda}\gamma_5 D_\alpha\chi_\beta$ terms.

$$S(\sigma,\lambda;e^a_\alpha,\chi_\alpha)=\int ed^2x[R\sigma-4i\epsilon^{\alpha\beta}/e\bar{\lambda}\gamma_5D_\alpha\chi_\beta-1/2g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma \\ -i/2\bar{\lambda}\gamma^\alpha\partial_\alpha\lambda+i\bar{\lambda}\gamma^\beta\gamma^\alpha\chi_\beta(\partial_\alpha\sigma-i/2\bar{\lambda}\chi_\alpha)] \tag{10}$$

This result has been obtained earlier in [13]. In agreement with general analyses of the Weyl anomaly and the reconstruction of the corresponding effective action [12], we suppose that the effective action has to be the sum of terms of the form: *Anomaly*(Weyl-inv.diff.op.)$^{-1}$*Anomaly*. Now, the conformally invariant operators, which should be the denominators in non-local expression for effective action, are defined as a variational derivatives of cocycle eq.(10) or what the same, of one of the Neveu-Schwarz action, because its differ by terms linear with respect to λ and σ. The cocycle eq.(10) is Weyl co-boundary of the effective action $W[e^a_\alpha,\chi_\alpha]$, which we are looking for.

$$S(\sigma,\lambda;e^a_\alpha,\chi_\alpha)=W[e^{\sigma/2}e^a_\alpha,e^{\sigma/4}(\chi_\alpha+\frac{1}{4}\gamma_\alpha\lambda)]-W[e^a_\alpha,\chi_\alpha] \tag{11}$$

The cocyclic property of the S

$$S(\sigma_1+\sigma_2,\lambda_1+\lambda_2;e^a_\alpha,\chi_\alpha)=S(\sigma_1,e^{-\sigma_2/4}\lambda_1;e^{\sigma_2/2}e^a_\alpha,e^{\sigma_2/4}(\chi_\alpha+\frac{1}{4}\gamma_\alpha\lambda_2))+S(\sigma_2,\lambda_2;e^a_\alpha,\chi_\alpha) \tag{12}$$

now is trivial consequence of previous relation. However, multiplying those denominators by anomaly and jointing them together, one doesn't reach desirable result, as easily to seen. Exploiting the fact that cocycle eq.(10) takes quadratic form on super-Weyl group variables, one could tries to perform gaussian integration in order to obtain the effective action. Unfortunately this procedure leads to complicated expression, which seems has in denominators differential operator degree greater than two. Apart from that, it is very hard to prove relation eq.(11) for that expression.

In order to avoid those difficulties let us proceed by following. First of all, let us notice that apart from curvature and the curl of Rarita-Schwinger field there are no terms with dimensions 2 and 3/2 whose are super Weyl invariant, satisfy to Wess-Zumino consistency condition and can be added to r.h.s. of eqs.(8) and (9) correspondingly . So as well as in ordinary 2d gravity there is no Weyl-invariant solutions of Wess-Zumino condition. Only term of dimension two can be considered

$$(\chi_\alpha\gamma^\beta\gamma^\alpha\chi_\beta)^2=(\frac{1}{2}\bar{\tilde{\chi}}_\alpha\tilde{\chi}^\alpha)^2=0 \tag{13}$$

Indeed, that term can be considered as the fourth degree of *spin* 3/2 part of Rarita-Schwinger field $\tilde{\chi}_\alpha=\gamma^\beta\gamma_\alpha\chi_\beta$ - quantity, which has only two independent components.

Let us now "improve" our anomaly expressions. It is impossible to add to such a local counterpart to R, in order to cancel it's σ-variation. Rather, it is possible to combine R in such a way to it's λ-variation vanish. Analogous improvement should to be done for super-Weyl anomaly. So the expression for anomaly, convenient for us takes form:

$$\mathcal{A}_2=R+i/2\nabla_\alpha(\bar{\chi}_\mu\gamma^\alpha\gamma^\beta\gamma^\mu\chi_\beta) \tag{14}$$
$$\mathcal{A}_{3/2}=-4i\frac{\epsilon^{\alpha\beta}}{e}\gamma_5D_\alpha\chi_\beta+D_\alpha(\gamma^\beta\gamma^\alpha\chi_\beta),$$

Let us notice, that "improved" anomaly expression also satisfy to Wess-Zumino condition, because improving terms can be represented as the Weyl-coboundaries of the local functionals.

$$\nabla_\alpha(\bar{\chi}_\mu \gamma^\alpha \gamma^\beta \gamma^\mu \chi_\beta)(x) = -1/4 \frac{\delta}{\delta\sigma(x)} \int e d^2 y \bar{\chi}_\beta \gamma^\beta D_\alpha \chi^\alpha \tag{15}$$

$$D_\alpha(\gamma^\beta \gamma^\alpha \chi_\beta)(x) = -2 \frac{\delta}{\delta\lambda(x)} \int e d^2 y [\bar{\chi}_\mu \gamma^\nu \gamma^\mu \gamma^\alpha D_\nu \chi_\alpha - i \bar{\chi}_\mu \gamma^\nu \gamma^\mu \chi_\nu \bar{\chi}_\alpha \chi^\alpha].$$

Now, multiplying $\mathcal{A}_2$ by inverse Laplace operator and by $\mathcal{A}_2$ we notice, that it's coboundary is local expression, as well as the coboundary of $\mathcal{A}_{3/2}(\mathcal{D})^{-1}\mathcal{A}_{3/2}$, here $\mathcal{D} = -i\gamma^\alpha D_\alpha + \chi_\alpha \bar{\chi}_\beta \gamma^\alpha \gamma^\beta$ is kinetic operator of spinor fields in Neveu- Schwarz action. However, we obtain the extra "improving" terms in our coboundary. Those are removed by adding the third non-local term to our action, which is constructed from remaining conformally-invariant operator. Finally the effective action takes the form

$$\begin{aligned} W[e,\chi] = \int [& e(R + i/2\nabla_\alpha(\bar{\chi}_\mu \gamma^\alpha \gamma^\beta \gamma^\mu \chi_\beta)) \frac{1}{e\Box} e(R + i/2\nabla_\alpha(\bar{\chi}_\mu \gamma^\alpha \gamma^\beta \gamma^\mu \chi_\beta)) + \\ & e^{3/4}(-4i\frac{\epsilon^{\alpha\beta}}{e}\gamma_5 D_\alpha \chi_\beta + D_\alpha(\gamma^\beta \gamma^\alpha \chi_\beta)) \frac{1}{e^{1/2}\mathcal{D}} e^{3/4}(-4i\frac{\epsilon^{\alpha\beta}}{e}\gamma_5 D_\alpha \chi_\beta + D_\alpha(\gamma^\beta \gamma^\alpha \chi_\beta)) - \\ & (R + i/2\nabla_\alpha(\bar{\chi}_\mu \gamma^\alpha \gamma^\beta \gamma^\mu \chi_\beta)) \frac{1}{e^{3/4}(i\gamma^\beta \gamma^\alpha \chi_\beta \partial_\alpha)} e^{3/4}\mathcal{A}_{3/2}] \end{aligned} \tag{16}$$

here $\Box$ is ordinary Laplace operator, acting on scalar fields. This is our desirable expression for the action of induced $2d$ supergravity, computed in supercoordinat-invariant regularization. The last term eq.(15) can be replaced by following local combination

$$\int e d^2 x [\frac{i}{8} \bar{\chi}_\beta \gamma^\beta D_\alpha \chi^\alpha + 2(\bar{\chi}_\mu \gamma^\nu \gamma^\mu \gamma^\alpha D_\nu \chi_\alpha - i \bar{\chi}_\mu \gamma^\nu \gamma^\mu \chi_\nu \bar{\chi}_\alpha \chi^\alpha)]. \tag{17}$$

The anomaly equations (8) and (9) remain unchanged under that replacement, so the non-locality related to the last term of the effective action (16) is removable. The addition of dimensionless actions - local countereterms like eq.(14) makes finite renormalization of the action. The overall constant in front of the action defined by the physical content of the theory.

The effective action for 2d supergravity, computed in super-Weyl invariant regularization scheme can be obtained from (15) by adding the non-super coordinate invariant action

$$S(\sigma,\lambda; e^a_\alpha, \chi_\alpha)|_{\sigma=\log e; \lambda=-2\gamma^\beta\chi_\beta} \tag{18}$$

accordingly to relation (11) it can be represented in manifest super-Weyl invariant form:

$$\tilde{W}[E^a_\alpha, \chi_\alpha] = W[e^{-1/2} e^a_\alpha, e^{-1/4} \gamma^\beta \gamma^\alpha \chi_\beta] \tag{19}$$

As easely to seen this action differs from eq.(16) by local expression' hence can be reached from that by finite renormalization, which will mean change of regularization scheme of the theory.

3 Conclusion and outlook

So, the main conclusion , that can be derived from our experience of work with Weyl anomaly may be formulated as following statement. The expressions acceptable as the conformal anomaly are following: the density of the Euler characteristic, Weyl-invariant lagrangian densities and variations of local functionals of appropriate dimensions (improving terms), those can be added to the action as the local counterterms. There already exists such the representative

of the class of solutions of the consistency condition, which has the linear and diagonal (in supersymmetric case) finite Weyl-variation. Corresponding cocycle will be quadratic. Integration of that cocycle by group variables is easily to perform in order to obtain the non-local expression for the effective action.

The arguments described above can be also applied to the four-dimensional supergravity. In that case, however, we cannot restore whole effective action as in $2d$, because the anomaly equations contain information only about anomalous dependence of the effective action from parameters of super Weyl group - trace of the metric and *spin* 1/2 part of the Rarita-Schwinger field. After taking into account manifest general covariance and local supersymmetry of the resulting expression there five components of the metric and eight components of Rarita-Schwinger field remain, dependence from which comes as an integration "constant" under integration of anomaly equations.

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